

Korea University

College of Information and Communication

CNCE 205: Discrete Mathematics

Midterm Examination: Apr 13, 2011

Name: ID:

1 (10)	
2 (5)	
3 (10)	
4 (15)	
5 (15)	
6 (15)	
7 (15)	
8 (15)	
9 (5) EXTRA	
Total (105)	

1. (a) Verify that $\neg p \wedge (\neg q \vee p) = \neg(p \vee q)$, not by making a truth table but by using known basic logical equivalences. 진리표를 만들지 말고 주어진 expression을 증명/도출하시오.

$\neg p \wedge (\neg q \vee p)$	
$= (\neg p \wedge \neg q)$ V $(\neg p \wedge p)$	Distributivity
$= (\neg p \wedge \neg q) \vee f$	Contradiction Rule
$= \neg p \wedge \neg q$	Contradiction Rule
$= \neg(p \vee q)$	De Morgan's Law

(b) Verify the following statement. 주어진 expression을 증명/도출하시오.

$A^c \cap (B^c \cup A) = (A \cup B)^c$.

$A^c \cap (B^c \cup A) = (A^c \cap B^c) \cup (A^c \cap A)$	Distributivity
$= (A^c \cap B^c) \cup$ Null	Null Rule
$= A^c \cap B^c$	Null Rule
$= (A \cup B)^c$	De Morgan's Law

2. Negate the statements. 다음 Statement들을 부정하시오.

(a) There exists $n \in Z$ such that for all $x \in R$, $x^n < 0$.

For all $n \in Z$, there exists $x \in R$ such that $x^n >= 0$.

(b) All good things come to an end.

There is a good thing that does not end.

3. Verify that the given argument form is valid. 다음 논리적 전개가 Valid 한지 가름하시오.

For all x $\in$ U, p(x) $\wedge$ q(x)
a $\in$ U
==========
p(a)

1. For all x $\in$ U, p(x) $\wedge$ q(x)	Given
2. a $\in$ U	Given
3. p(a) $\wedge$ q(a)	Principle of Specification
4. Therefore, p(a)	In Particular

4. Prove or disprove the following assertions. Indicate the proof technique used. 다음을 읽고 증명을 하거나 부정을 하시오. 증명/부정 방법을 말하시오.

(a) Let A and B be sets. Show: If $A \subseteq B^c$ then $A \cap B = \varphi$

Suppose $A \cap B$ is not empty.
Then we have some $x \in A \cap B$. Thus, $x \in A$ and $x \in B$.
Since $x \in A$ and $x \notin B^c$, it follows $A \not\subseteq B^c$

QED/ Proof with a contrapositive.

(b) Let R be a relation on real numbers, such that $\mathbf{x\ R\ y}$ if and only if $x > y - 1$. Prove or disprove that R is reflexive, symmetric and transitive.

Reflexive: For all x, x > x -1 (direct)

Not symmetric: 1 R 0 but 0 R 1 does not hold (counter example)

Not transitive: 0 R 1/2 and 1/2 R 1 but 0 R 1 does not hold (counter example)

(c) Let X be the set of real numbers and define a relation R on X by, $\boldsymbol{f\ R\ g}$ if and only if f(0) <= g(0). Prove that R is not a partial order relation.

R is not anti-symmetric.
For the functions f(x) = x and g(x) = -x, f(0) = 0 = g(0) but f ~= g.
(counter example)

5. The following is an algorithm to convert a positive integer *m* to the base *b* integer $c_n c_{n-1} \ldots c_1 c_0$ (정수 m 을 b 진수로 나타내는 알고리즘이다).

Input: m, b
Output: c, n

```
dec_to_base_b (m, b, c, n) {
        n = -1
        while (m > 0) {
                n = n+1              // LSB first
                c_n = m mod b        // remainder
                m = ⌊m/b⌋            // quotient
        }
}
```

(a) What is the input that makes this algorithm run with the worst complexity? (이 알고리즘이 최악의 경우로 실행 되게 하는 입력값은 무엇인가?)

(b) What is the worst time algorithm complexity in terms of n? (n을 인자로 표현 한 최악의 경우의 Complexity는 무엇인가?)

6. What is the tight bound for the complexity function: $\log(\sum_{i=1}^{n} i)$. 주어진 함수의 Tight Bound를 구하시오.

(1)

$\log (1 + 2 + \dots + n) \leq \log (n + n + \dots + n) = \log (n \cdot n) = \log n^2 = 2\log n$

$O(\log n)$ for all $n \geq 1$

(2)

$\log (1 + 2 + \dots + n) \geq \log(\lceil n/2 \rceil + \dots + (n-1) + n)$

$\geq \log (\lceil n/2 \rceil + \dots + \lceil n/2 \rceil + \lceil n/2 \rceil)$

$= \log(\lceil (n+1)/2 \rceil \lceil n/2 \rceil)$

$\geq \log((n/2)(n/2))$

$= \log(n^2/4) = 2(\log n) - \log 4$

$= \log n + \log n - 2 > \log n$ for all $n \geq 4$

$\Omega(\log n)$

Therefore, $\Theta(\log n)$

7. An association list is a list of correspondences of items from one set with items of another. Write a recursive algorithm, called AL, that constructs an association list between two lists. You can assume the number of elements of each input list is always the same. If needed, use the functions below (like First and etc.) for devising your algorithm.

어떤 두 같은 길이의 리스트가 주어지면 순서대로 위에 표현 한 대로 짝을 지어 Association List 를 만드는 **recursive 한 알고리즘**을 (이름: AL) 짜시오. 필요하면 아래 First 등의 함수를 쓸 수 있다.

Example input and output of AL:

AL ([x y z] [1 2 3]) → [[x 1] [y 2] [z 3]]
AL (null null) → null
AL ([1] [a b]) → such input is not possible in the first place
(애초에 이러한 입력은 불가능 가정)

Functions you can use:

First(L) → returns first element of list L
Rest(L) → returns rest of list L
Listp(i) → returns whether i is list or not (true or false)
Nullp(i) → returns whether i is null or not (true or false)
Insert (p, L) → inserts pair p into list L
e.g. Insert ([a b], [[c d] [e f]] → [[a b] [c d] [e f]]
Pair-list (x y) → returns a pair list composed of x and y
e.g. Pair-list (x y) → [x y]

```
AL (L1, L2) {
if Nullp(L1) then return null;
else if Nullp(L2) then return null;
else return Insert ( pair-list( first(L1), first(L2) ),
                     AL( rest(L1), rest(L2)) )
}
```

8. (a) Let L be the set of all strings, including the null string λ, that can be constructed by repeated application of the following rules (L 은 다음 규칙에 의해 만들어 질 수 있는 모든 스트링이다 (null 스트링 포함), 이 때 baabab ∈ L를 보이시오):

A. If $\alpha \in L$, then $a\alpha b \in L$ and $b\alpha a \in L$.
B. If $\alpha \in L$ and $\beta \in L$, then $\alpha\beta \in L$.

Show that baabab ∈ L.

Ans)

$\lambda \in L$ (Given)
$\Rightarrow b\lambda a \in L$ (Rule A)
$\Rightarrow ba \in L$.

Similarly, $ab \in L$.
$\Rightarrow ba\ ab\ ab \in L$ (Rule B)

(b) Using matrix representation and computation, determine if R = {(w,w), (w,y), (x,x), (y,y), (z,z)} is a partial order. 행렬을 이용하여 R 이 Partial order 임을 보이시오.

Ans) It is a partial order. In matrix representation,

$$R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since the main diagonal is all 1, R is reflexive. And because R(1,2) != R(2,1), it is antisymmetric. Finally,

$$R^2 = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since all entries in R corresponding to the non-zero entries in R^2 are also non-zero, it is transitive. Therefore, R is a partial order.

9. [EXTRA] Cite examples of computer systems (software or hardware) which use the following concepts of discrete mathematics you have learned in Chapter 1 through 4. 1과에서 4과까지 배운 내용들이 어떤 컴퓨터 소프트웨어나 하드웨어 시스템에 적용 가능 한가?

(a) Logic, rules of inference, mechanical proof

Data mining, program correctness, data base, AI

(b) Relation

Data base systems

(c) Algorithm complexity (one hint: what might we be able to do with algorithms with extremely high complexity?)

Cryptography

(d) Recursion

AI, Fractal art, automatic music composition, …