이산수학 중간고사

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학번 _____ 이름 _____

1. Verify using mathematical induction that for all positive integers n, $\frac{1}{2^2-1}+\frac{1}{3^2-1}+\frac{1}{4^2-1}+\cdots+\frac{1}{(n+1)^2-1}=\frac{3}{4}-\frac{1}{2(n+1)}-\frac{1}{2(n+2)}$. [5점]

Sol)

2. Let p : "The bug is in module 17", q : "The bug is in module 81", r : "The bug is a numerical error". From the following propositions, prove "The bug is in module 17." Specify what inference rule you used in each step.[6점]

The bug is either in module 17 or in module 81.

The bug is a numerical error.

If the error is a numerical error, it is not in module 81.

Sol)

3. What would the following Prolog input return as output when you query **uncle(cheolsu, hoduk)**? _____ [3점]

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father(cheolsu, youngduk).

father(youngsu, youngduk).

father(youngduk, youngho).

father(hoduk, youngho).

male(hoduk).

male(youngho).

brother(X, Y):-male(Y), father(X, P), father(Y, P), X<>Y.

uncle(X,Y):-father(X,P), brother(P,Y).
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4. Determine whether the following relations defined on the set of positive integers are reflexive, symmetric, anti-symmetric, transitive, and/or partial order. [각 3점]

   A. $(x,y) \in R$ if $(x-y)$ *is divided by* 3. _____

   B. $(x,y) \in R$ if $|x-y| = 2$. _____

   C. $(x,y) \in R$ if $x \geq y$. _____

5. We want to compare the complexities of the two functions $n!$ and 2^n by computing $\Theta(\lg n!)$.

   A. What is $\Theta(\lg n!)$? [10점]

Sol)

   B. So which is more complex between $n!$ and 2^n ? [3점]

Sol)

6. In RSA, suppose we choose two prime numbers as p=5, q=7 so that n=qp = 35 and $\phi = (p-1)(q-1) = 24$. And we publish the public key (n,e)=(35,5). [각 7점, 총21점]

   A. What is the private key corresponding to the public key? [Hint: first get the inverse modulo d of e, which satisfies de = 1 (mod ϕ) and $0 \le d < \phi$, using the inverse Euclid algorithm.]_____

   B. What is the cyphertext of message m=12? _____

   C. Show that the RSA algorithm successfully recovers m=12 from the cyphertext.

7. A company buys computers from 3 vendors and tracks the number of defective machines. The following table shows the results.

	Vendor		
	Acme	*DotCom*	*Nuclear*
Percent purchased	*55*	*10*	*35*
Percent defective	*1*	*3*	*3*

Let A denote the event "the computer was purchased from Acme," let D denote the event "the computer was purchased from Dot-Com," let N denote the event "the computer was purchased from Nuclear," and let B denote the event "the computer was defective." **Find P(A|B).** [15점]

Sol)

8. Find the number of integer solutions of $x_1 + x_2 + x_3 = 15$, subject to the condition $x_1 = 1, x_2 \geq 0, x_3 \geq 0$. [5점]

Sol)

9. An inventory consists of a list of 100 items, each marked "available" or "unavailable". There are 55 available items. Show that there are at least two available items in the list exactly nine items apart. [7점]

Sol)

10. Show that the number of disk movements in the Hanoi Towers algorithm has $O(2^n)$ complexity using recurrence relation. [9점]

Sol)

11. Assume that the deer population of Rustic County is 100 at time n=0 and 170 at time n=1 and that the increase from time n-1 to time n is three times the increase from time n-2 to n-1.

A. Predict the deer population at time n. [5점]

Sol)

B. What is the population at n=5? [2점]

Sol)