HW 2 Solutions

October 12, 2012

5-1

30.
Since $dc|nc$, $nc = dcq$ for some integer q. Since dc is a divisor of nc, $c \neq 0$. Therefore, we may cancel c in $nc = dcq$ to obtain $n = dq$. Therefore, $d|n$.

33. $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 + 1 = 30031 = 509 \cdot 59$

5-2

24. 1000100010

33. 22, 3D, DF, 190, 400, 3034

39. 130FF7

5-3

11. $s = -64$, $t = 93105$.

35. $s = 3$

38. $s = 67$

5-4

3. `WEISERKYEIEFTKK_`, where _ is the space character.

11. $a = c^s \bmod z = 250^{159} \bmod 391 = 10$

14. $s = 3961$

6-2

20. First, line up the Vesuvians and the Jovians. This can be done in 18! ways. For each of these arrangements, we can place the Martians in 5 of the 19 in-between and end positions, which can be done in $P(19, 5)$ ways. Thus there are $18! \cdot P(19, 5)$ arrangements.

32. The solution to Exercise 31 shows that there are 838 different pizzas. The total number of ways that 4 pizzas can be selected is $C(838, 4) + 3C(838, 3) + 3C(838, 2) + C(838, 1)$.

41. Six: 00011111, 10001111, 11000111, 11100011, 11110001, 11111000

61. $C(10, 3)$

67. $C(46, 2)C(4, 2)$

78. Fix a starting position, and move around the table. When a handshake begins, write R; when a handshake ends, write a U. The result is a sequence of n R's and n U's in which the number of R's is always greater than or equal to the number of U's. Furthermore, the correspondence between such sequences and handshakes is one-to-one and onto. Since the number of sequences of n R's and n U's in which the number of R's is always greater than or equal to the number of U's is C_n (see Example 6.2.23), the number of ways that $2n$ persons seated around a circular table can shake hands in pairs without any arms crossing is also C_n.

6-3

12. 4^{100}

19. $C(8+2-1,8)$

27. There are $C(14+3-1,14)$ solutions satisfying $0 \leq x_1$, $1 \leq x_2$, $0 \leq x_3$. Of these, $C(8+3-1,8)$ have $x_1 \geq 6$;$C(6+3-1,6)$ have $x_2 \geq 9$; and there is one with $x_1 \geq 6$ and $x_2 \geq 9$. Thus there are

$$C(8+3-1,8)+C(6+3-1,6)-1$$

solutions with $x_1 \geq 6$ or $x_2 \geq 9$. Therefore there are

$$C(14+3-1,14)-[C(8+3-1,8)+C(6+3-1,6)-1]$$

of the desired type.

40.$C(20,5)C(15,5)$

6-5

25.

$$\frac{1}{C(49,6)}$$

47. Suppose that behind the first two doors are goats, and behind the third door is the car. Consider your three initial choices. If you initially choose door one, your switch will move you to the door with the car, and you win. Similarly, if you initially choose door two, your switch will move you to the door with the car, and you win. However, if you initially choose door three, your switch will move you to a door with a goat, and you lose. Therefore, the probability of winning the car is $\frac{2}{3}$.

54. There are four ways that the sequence of tosses can start: TT, HT, TH, HH. If the sequence starts TT, the second player wins. If the sequence starts HT, the first player wins. If the sequence begins TH or HH, the first T after

the first two coin tosses must be preceded by H and the first player wins. Thus the probability that the first player wins is 3/4. It's best to be the first player!

6-6

12. Let E_1 be the event "sum of 6," let E_2 be the event "doubles," and let E_3 be the event "at least one 2." We want

$$P(E_1 \cup E_2|E_3) = \frac{P((E_1 \cup E_2) \cap E_3)}{P(E_3)}.$$

The event $(E_1 \cup E_2) \cap E_3$ comprises (2,4), (4,2), (2,2); therefore,

$$P((E_1 \cup E_2) \cap E_3) = 3\left(\frac{1}{12}\right)^2.$$

The solution to Exercise 11 shows that

$$P(E_3) = \frac{23}{144}.$$

Therefore,

$$P(E_1 \cup E_2|E_3) = \frac{P((E_1 \cup E_2) \cap E_3)}{P(E_3)} = \frac{3/144}{23/144} = \frac{3}{23}.$$

18. No

41. 253

54. Yes.
Since E and F are independent, $P(E \cap F) = P(E)P(F)$. Since $E \cap F$ and $E \cap \bar{F}$ are mutually exclusive and $E = (E \cap F) \cup (E \cap \bar{F})$,

$$P(E) = P(E \cap F) + P(E \cap \bar{F}).$$

Now,

$$P(E \cap \bar{F}) = P(E) - P(E \cap F) = P(E) - P(E)P(F) = P(E)[1 - P(F)] = P(E)P(\bar{F}).$$

Therefore, E and $\bar{F}$ and independent.

6-7

8. $C(5, 2)$

12. $C(12 + 3 - 1, 12) + C(11 + 3 - 1, 11) + C(10 + 3 - 1, 10)$

6-8

9. Professor Euclid is paid 26 times per year. Since there are 12 months, by the Pigeonhole Principle, at least $\lceil 26/12 \rceil = 3$ of them will be assigned the same first and last names.

12. Let a_i denote the position of the ith available item. The 110 numbers

$$a_1, \ldots, \ a_{55}; \quad a_1 + 9, \ldots, \ a_{55} + 9$$

have values between 1 and 109. By the second form of the Pigeonhole Principle, two must coincide. The conclusion follows.

31. Suppose that the numbers around the circle are $x_1, \ldots, x_{12}$. We argue by contradiction. Suppose that

$$\begin{aligned}
x_1 + x_2 + x_3 &\leq 19 && (1)\\
x_2 + x_3 + x_4 &\leq 19 && (2)\\
&\vdots && (3)\\
x_{10} + x_{11} + x_{12} &\leq 19 && (4)\\
x_{11} + x_{12} + x_1 &\leq 19 && (5)\\
x_{12} + x_1 + x_2 &\leq 19. && (6)
\end{aligned}$$

Summing, we obtain the contradiction

$$234 = 3\left(\frac{12 \cdot 13}{2}\right) = 3(x_1 + \cdots + x_{12}) \leq 12 \cdot 19 = 228.$$