Answers to HW 1

Discrete Math

September 13, 2012

1-2.

14. No heads or no tails were obtained.
54. $(p \vee q) \wedge \neg p$
65. $\neg(p \vee q \vee \neg r)$

1-3.

48. $q \rightarrow (p \vee r)$
68. $P \not\equiv Q$
72. Shirley will not take the bus and not catch a ride to school.

1-4.

15. Valid
19. Modus ponens
22.
p: "there is gas in the car"
q: "I go to the store"
r: "I get a soda"
s: "the car transmission is defective"
Hypotheses are:

$$p \rightarrow q,\ q \rightarrow r, \neg r$$

$$\begin{aligned} (p \rightarrow q) \wedge (q \rightarrow r) &\Rightarrow p \rightarrow r \text{ (hypothetical syllogism)} \\ (p \rightarrow r) \wedge \neg r &\Rightarrow \neg p \text{ (modus tollens)} \\ \neg p &\Rightarrow \neg p \vee s \text{ (addition)} \end{aligned}$$

Since $\neg p \vee s$ represents the proposition "there is not gas in the car or the car transmission is defective," we conclude that the conclusion does follow from the hypotheses.

1-5.

45. $\exists x(P(x) \wedge Q(x))$
46. $\forall x(Q(x) \rightarrow P(x))$
59. Literal meaning: Every hospital did not report every month. (Domain of discourse: the 74 hospitals)
Intended meaning (most likely): Some hospital did not report every month. Let $P(x)$ denote "x is a hospital" and $Q(x)$ denote "x reports every month." The intended statement is $\exists x(P(x) \wedge \neg Q(x))$.
69. Let
$p(x)$: x is good.
$q(x)$: x is too long.
$r(x)$: x is short enough.
The domain of discourse is the set of movies. The assertions are
$\forall x(p(x) \rightarrow \neg q(x))$
$\forall x(\neg p(x) \rightarrow \neg r(x))$
$p(Love\ Actually)$
$q(Love\ Actually)$.

1-6.

12. Someone is taller than or the same height as everyone.
35. $\forall x \exists y E(x) \rightarrow A(x, y)$
58. True

2-1.

14. Let x and y be rational numbers. Then there exist integers m_1, n_1, m_2, n_2 such that $x = m_1/n_1$ and $y = m_2/n_2$. Now $xy = (m_1 m_2)/(n_1 n_2)$. Therefore, xy is rational.
57. True. We first prove $A \subseteq B$. Let $x \in A$. We divide the proof into two cases. First, we consider the case that $x \in C$. ~~Then $x \notin C$~~. Then $x \notin A \triangle C$. Therefore $x \notin B \triangle C$. Then $x \in B$ (since if $x \notin B$, then $x \in B \triangle C$. Next, we consider the case that $x \notin C$. Then $x \in A \triangle C$. Therefore, $x \in B \triangle C$. Thus $x \in B$. In either case, $x \in B$, and so $A \subseteq A$, and so $A = B$.



2-2.

8. Suppose, by way of contradiction, that $x + y$ is rational. Since x and $x + y$ are rational, there exist integers p_1, p_2, q_1, q_2 such that $x = p_1/q_1$ and $x + y = p_2/q + 2$. Now

q2

$$y = (x + y) - x = \frac{p_2}{q_2} - \frac{p_1}{q_1} = \frac{p_2 q_1 - p_1 q_2}{q_1 q_2}$$

Therefore y is rational, which is a contradiction.
24. True. Suppose that for every j, $s_j \leq A$. Since $s_j \leq A$ for all j and $s_i < A$,

$$s_1 + \cdots + s_i + \cdots + s_n < A + \cdots A + \cdots + A = nA$$

Dividing by n we obtain

$$\frac{s_1 + \cdots + s_n}{n} < A$$

which is a contradiction.
40.

$$\begin{aligned} max(x, y) + min(x, y) &= \frac{x + y + |x - y|}{2} + \frac{x + y - |x - y|}{2} \\ &= \frac{x + y + |x - y| + x + y - |x - y|}{2} \\ &= \frac{2x + 2y}{2} = x + y \end{aligned}$$

2-3.

4.

1. $\neg p \vee t$
2. $\neg q \vee s$
3. $\neg r \vee s$
4. $\neg r \vee t$
5. $p \vee q \vee r \vee u$
6. $t \vee q \vee r \vee u$ (from 1.5)
7. $s \vee t \vee r \vee u$ (from 2.6)
8. $s \vee t \vee u$ (from 3.7)

6. $(p \leftrightarrow r) \equiv (p \rightarrow r)(r \rightarrow p) \equiv (\neg p \vee r)(\neg r \vee p)$

1. $\neg p \vee r$
2. $\neg r \vee p$
3. r
4. p (from 2.3)

8.

1. $a \vee \neg b$
2. $a \vee c$
3. $\neg a$
4. $\neg d$
5. b (negated conclusion)
6. $\neg b$ (from 1.3)

Now 5 and 6 combine to give a contradiction.

2-4.

2.

$$1\cdot 2+2\cdot 3+\cdots+n(n+1)+(n+1)(n+2) = \frac{n(n+1)(n+2)}{3}+(n+1)(n+2) = \frac{(n+1)(n+2)(n+3)}{3}$$

22. Assume that $11^n - 6$ is divisible by 5.

$$11^{n+1} - 6 = 11^n \cdot 11 - 6 = 11^n(10+1) - 6 = 10 \cdot 11^n + 11^n - 6$$

which is divisible by 5.