3-1

50.
(a) $f \circ f = \{(a,a),\ (b,b),\ (c,a)\}$, $f \circ f \circ f = \{(a,b),\ (b,a),\ (c,b)\}$
(b) $f^9 = f$, $f^{623} = f$.

74.
(You should use the definition above Prob. 70)
If $x \in X$, then $x \in f^{-1}(f(\{x\}))$. Thus $\cup\{S|S \in \mathcal{S}\} = X$. Suppose

$$a \in f^{-1}(\{y\}) \cap f^{-1}(\{z\})$$

for some $y, z \in Y$. Then $f(a) = y$ and $f(a) = z$. Thus $y = z$. Therefore $\mathcal{S}$ is a partition of X.

100.
False. A counterexample is $x = 2$, $y = 2.6$.

3-2

113.

$$\sum_{i=0}^{n-1} C_i C_{n-i-1}$$

131.
f is one-to-one. Suppose $f(\alpha) = f(\beta)$. We then have $\alpha\alpha^R = \beta\beta^R$. Now the first half of the first string, namely α, must equal the first half of the second string, namely β. Therefore f is one-to-one.
Notice $|f(\alpha)|$ is an even integer for all $\alpha \in X^*$. Therefore, $f(\alpha) \neq aaa$ for all $\alpha \in X^*$. Thus f is not onto.

133.
Taking $\alpha = \lambda$, the first rule tells us that $a\alpha b = ab \in L$ and $b\alpha a = ba \in L$.

Taking $\alpha = ba$ and $\beta = ab$, the second rule tell us that $\alpha\beta = baab \in L$. Finally, taking $\alpha = baab$ and $\beta = ab$, the second rule tells us that $\alpha\beta = baabab \in L$.

3-3

31.
Symmetric.

34.
Reflexive, symmetric.

59.
It may be the case that for $x \in X$, there is no $y \in X$ such that $(x, y) \in R$. Consider, for example, $X = \{1, 2, 3\}$, $R = \{(1, 1), (2, 2), (1, 2), (2, 1)\}$, and $x = 3$.

3-4

19.
$\{(i, j)|i, j \in \{1, 2, 3, 4\}\}$, $[1] = [2] = [3] = [4] = \{1, 2, 3, 4\}$.

36.
(a) Reflexive: $(x, y)R(x, y)$, Symmetric: $(x, y)R(x', y') \Rightarrow (x', y')R(x, y)$ for all three cases, Transitive: $(x, y)R(x', y') \wedge (x', y')R(x'', y'') \Rightarrow (x, y)R(x'', y'')$ for all nine cases
(b) Cylinder.

42.
Suppose $[x] = [y]$. Then xRy. Therefore, $g(x) = g(y)$.

3-5

12.
Symmetric and transitive.

3-6

15.
TEMP1 := BUYER[Name=United Supplies] TEMP2 := TEMP1[TEMP1.Part No = SUPPLIER.Part No] SUPPLIER TEMP3 := TEMP2[Part No, Amount]

Part No	Amount
335B2	220
2A	14
1199C	296
772	39

4-2

5.
Let ϕ denote the empty position. Then,
$(34) \rightarrow (\phi, 34) \rightarrow (20, 34) \rightarrow (20, \phi, 34) \rightarrow (\phi, 20, 34) \rightarrow (19, 20, 34) \rightarrow (19, 20, \phi, 34) \rightarrow (19, \phi, 20, 34) \rightarrow (\phi, 19, 20, 34) \rightarrow (5, 19, 20, 34)$

4-3

17. $\Theta(n)$
27. $\Theta(n)$
34. For $n \geq 1$,

$$2^n = 2 \cdot 2^{n-1} \leq 2 \cdot (2 \cdot 3 \cdots n) = 2n!$$

4-4

5.
Since $n \neq 1$ and $n \neq 2$, we execute the line

return $robot_walk(n-1) + robot_walk(n-2)$

with $n = 4$. When we compute $robot_walk(3)$, since $n \neq 1$ and $n \neq 2$, we execute the line

return $robot_walk(n-1) + robot_walk(n-2)$

with $n = 3$. The algorithm returns the value 2 for $robot_walk(2)$ and the value 1 for $robot_walk(1)$. Therefore for $n = 3$, the algorithm returns the value

$robot_walk(2) + robot_walk(1) = 2 + 1 = 3$

Therefore for $n = 4$, the algorithm returns the value

$robot_walk(3) + robot_walk(2) = 3 + 2 = 5$.

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