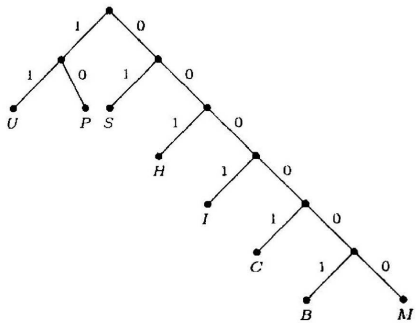
HW 5 Solutions

December 4, 2012

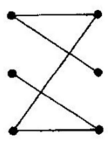
9-1

25.



9-4

3.



9-5

12.
Number of vertices $= mi + 1$, number of terminal nodes $= (m-1)i + 1$

17.
Proof by induction n:
(basis) $n = 1$: The tree consists of 3 vertices: root and two children. Thus $I = 0$, $E = 2$ and $E = 2 = 0 + 2 \cdot 1 = I + 2n$.
(induction hypothesis) assume that the equation is true for n.
(induction step) let T be a tree with $n + 1$ internal vertices. Let T' be the tree obtained from T by deleting two sibling terminal vertices and the edges incident on them. Let p denote the (former) parent of the deleted siblings. The resulting tree T' has n internal vertices. let I' and E' denote the internal and external path lengths for T'. By the inductive assumption, $E' = I' + 2n$. If len denotes the length of the simple path from the root to p in T, the external path length in T is obtained from the external path length in T' by adding $2(len + 1)$, to account for the two new paths to the children of p, and by subtracting len, to account for the loss of the path to the former terminal p; thus,

$$E = E' + 2(len + 1) - len = I' + 2n + len + 2$$

The internal path length in T' is obtained from the internal path length in T by subtracting len to account for the loss of the path to p; thus

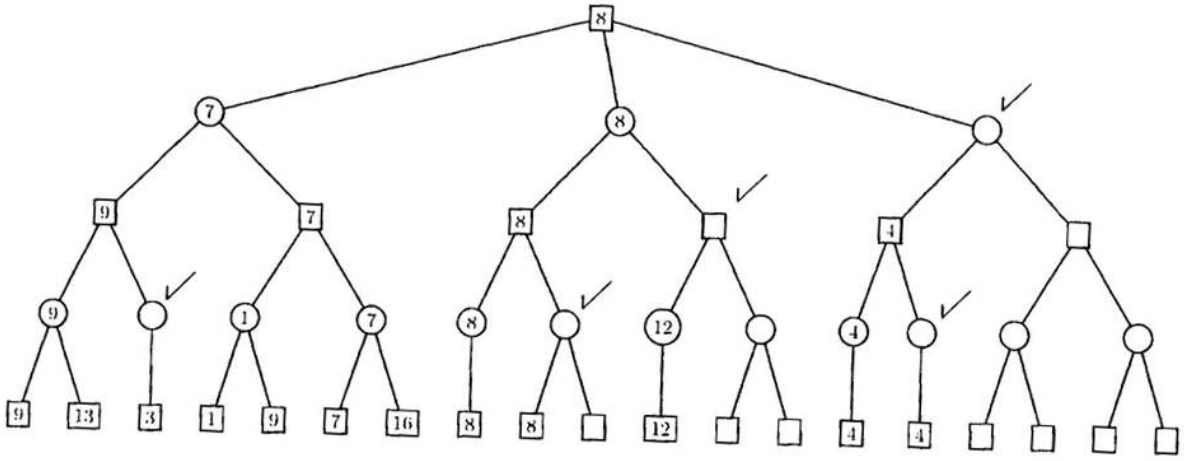
$$E = I' + 2n + len + 2 = I - len + 2n + len + 2 = I + 2(n + 1)$$

9-6

36.
First, note that any subset of $n - 1$ vertices is a vertex cover. Second, note that any subset V' of $n - 2$ edges is not a vertex cover. [If v and w are distinct vertices not in V', then edge (v, w) violates the condition that either v or w is in V'.]

9-9

24.



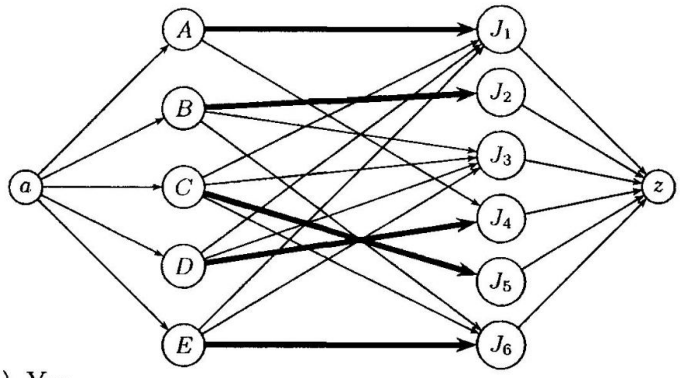
10-2

12.
(a,b)-10, (a,f)-11, (a,j)-1, (b,c)-6, (b,g)-9, (f,b)-5, (f,g)-6, (j,g)-1, (c,d)-8, (c,h)-2, (g,c)-4, (g,h)-12, (d,e)-8, (h,i)-6, (h,n)-6, (h,m)-2, (m,n)-2, (e,z)-8, (i,z)-6, (n,z)-8. All other edges have flow equal to 0.

10-4

8.

(a)(b)



(c) Yes

11-3

3. If $(S, +, \cdot, ', 1, 8)$ is a Boolean algebra, then $x + x' = 1$. In this case, we must have

$$lcm(x, 8/x) = 8, \quad \text{for } x = 1, 2, 4, 8$$

However, for $x = 4$, $lcm(4, 8/4) = lcm(4, 2) = 4$. Therefore, this system is not a Boolean algebra.

11-4

18. $\overline{xy}z \vee x\overline{yz} \vee \overline{xyz} \vee \overline{x}yz$

11-5

7. $xy = (x \uparrow y) \uparrow (x \uparrow y)$
18. $x \downarrow y = ((x \uparrow x) \uparrow (y \uparrow y)) \uparrow ((x \uparrow x) \uparrow (y \uparrow y))$