Quiz 2 Solutions

October 12, 2012

1.

$$\sum_{k=0}^{n-1}(k+1)^2 r^{n-k-1}$$

2. Let us denote the matrix representation of the given relation by A. In order to show that the relation is transitive, we should show that all non-zero entries in A^2 should have a corresponding non-zero entry in their positions. Now,

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

and

$$A^2 = \begin{pmatrix} 2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 0 & 2 & 0 \\ 1 & 0 & 2 & 1 \end{pmatrix}$$

If we compare the non-zero entries of A^2 and A, we see A^2 has one more entry. Therefore, the relation represented by A is not transitive.

3. 3: $[1] = [3] = [5] = \{1, 3, 5\},\ [2] = [6] = \{2, 6\},\ [4] = \{4\}$

4.

- Reflexive: $(A, A) \in R$ since $A \subseteq A$ for all A.
- Not symmetric: if $A \subset B$, $(A, B) \in R$ since $A \subseteq B$, but $(B, A) \notin R$ because $B \not\subseteq A$.
- Antisymmetric: since if $A \subseteq B$ and $B \subseteq A$ then $A = B$.
- Transitive: if $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.
- Not partial order: because it is not antisymmetric.

5.

- One-to-one: since $f(\alpha) = f(\beta)$ it means $\alpha\alpha = \beta\beta$, which is impossible unless $\alpha = \beta$.
- Not onto: $|f(\alpha)|$ is an even integer. Therefore, f does not map to any odd length strings.

6. See proof on p. 203.

7.
Note that $2^n = 2 \cdot 2^{n-1} \leq 2 \cdot (2 \cdot 3 \cdots n) = 2n!$. Namely, $2^n \leq 2n!,\ n \geq 1$. Therefore, $2^n = O(n!)$.

8.
The number of rabbits is described by equations $a_1 = 1,\ a_2 = 2,\ a_n = a_{n-1} + a_{n-2}$. Notice that it is the same as the Fibonacci sequence except for the initial terms. In fact, $a_n = \{f_{n+1}\}$. The question is about a_{12}. It is $f_{13} = 228$.

9. bab

10. If you give $n < 0$ as input, the program never terminates. An algorithm must terminate.