# Answers to HW 1

## Discrete Math

## September 24, 2013

**Section 1.2**

13. No heads were obtained.

53. $p \vee q$

64. $\neg p \wedge \neg q \wedge r$

**Section 1.3**

49. $(q \wedge p) \rightarrow \neg r$

67. $P \not\equiv Q$

71. Either Dale is not smart or not funny.

**Section 1.4**

13. Valid

20. Disjunctive syllogism

23.

Let *p* denote the proposition "Jill can sing," let *q* denote the proposition "Dweezle can play," let *r* denote the proposition "I'll buy the compact dist," and let *s* denote the proposition "I'll buy the compact dist player." Then the hypotheses are :

$$(p \vee q) \rightarrow r,\ p,\ s.$$

From *p*, we may use addition to conclude $p \vee q$. From $p \vee q$ and $(p \vee q) \rightarrow r$, we may use modus ponens to conclude *r*. From *r* and *s*, we may use conjunction to conclude $r \wedge s$. Since $r \wedge s$ represents the proposition "I'll buy the compact disk and the compact disk player," we conclude that the conclusion does follow from the hypotheses.

**Section 1.5**

45. $\exists x(P(x) \wedge Q(x))$

46. $\forall x(Q(x) \rightarrow P(x))$

58.

Literal meaning: Every old thing does not covet a twenty-something. Intended meaning: Some old thing does not covet a twenty-something. Let *P(x)* denote the statement "*x* is an old thing" and *Q(x)* denote the statement " *x* covets a twenty-something." The intended statement is $\exists x(P(x) \wedge \neg Q(x))$.

72.

Let P(x) denote the propositional function "x is a member of the Titans," let Q(x) denote the propositional function " x can hit the ball a long way," and let R(x) denote the propositional function "x can make a lot of money." The hypotheses are

$$P(Ken), Q(Ken), \forall x\, Q(x) \rightarrow R(x)$$

By universal instantiation, we have Q(Ken) $\rightarrow$R(Ken). From Q(Ken) and Q(Ken) $\rightarrow$R(Ken), we may use modus ponens to conclude R(Ken). From P(Ken) and R(Ken), we may use conjunction to conclude P(Ken) $\wedge$ R(Ken). By existential generalization, we have $\exists x P(x) \wedge R(x)$ or, in words, someone is a member of the Titans and can make a lot of money. We conclude that the conclusion does follow from the hypotheses.

**Section 1.6**

11. Everyone is taller than or the same height as someone.

34. $\forall x \neg A(x, Profesor\ Sandwich)$

59. True

**Section 2.1**

15. Let x be a nonzero rational number. Then there exist integers $m \neq 0$ and $n \neq 0$ such that $x = m/n$. Now $1/x = n/m$. Therefore $1/x$ is rational.

56.

By definition

$$(A \triangle B) \triangle A = [(A \triangle B) \cup A] - [(A \triangle B) \cap A]$$

Show that

$$(A \triangle B) \cup A = A \cup B \text{ and } (A \triangle B) \cap A = A \cap \overline{B}$$

The statement then follows easily.

**Section 2.2**

9. False; a counterexample is $x = 0, y = \sqrt{2}$.

23.

The statement is false. $Let\, s_1 = s_2 = 3$. Then A = 3. For no *i* do we have $s_i > A$. The proof is by counterexample.

39.

Suppose that $x \geq y$. Then

$$\min\{x,y\} = y \quad \text{and} \quad |x-y| = x-y.$$

$$Thus$$

$$\min\{x,y\} = y = \frac{2y}{2} = \frac{x+y-(x-y)}{2} = \frac{x+y-|x-y|}{2}.$$

$$The\, other\, case\, is\, x < y.\, Then$$

$$\min\{x,y\} = x \quad \text{and} \quad |x-y| = y-x.$$

$$Thus$$

$$\min\{x,y\} = x = \frac{2x}{2} = \frac{x+y-(y-x)}{2} = \frac{x+y-|x-y|}{2}.$$

**Section 2.3**

3. 1. $\neg p \vee r$

   2. $\neg r \vee q$

   3. $p$

   4. $\neg p \vee q$ from 1,2

5. $q$ from 3,4

6. $(p \leftrightarrow r) \equiv (p \rightarrow r)(r \rightarrow p) \equiv (\neg p \vee r)(\neg r \vee p)$

1. $\neg p \vee r$

2. $\neg r \vee p$

3. $r$

4. $p$ from 2,3

8. 1. $a \vee \neg b$

2. $a \vee c$

3. ¬a

4. ¬d

5. $b$ negated conclusion

6. ¬b from 1,3

**Section 2.4**

3. $1(1!) + 2(2!) + \cdots + n(n!) + (n+1)(n+1)!$
$= (n+1)! - 1 + (n+1)(n+1)! = (n+2)! - 1$

23.

Suppose that 4 divides $6 \cdot 7^n - 2 \cdot 3^n$. Now

$$\begin{aligned} 6 \cdot 7^{n+1} - 2 \cdot 3^{n+1} &= 7 \cdot 6 \cdot 7^n - 3 \cdot 2 \cdot 3^n \\ &= 6 \cdot 7^n - 2 \cdot 3^n + 6 \cdot 6 \cdot 7^n - 2 \cdot 2 \cdot 3^n \\ &= 6 \cdot 7^n - 2 \cdot 3^n + 36 \cdot 7^n - 4 \cdot 3^n \end{aligned}$$

Since 4 divides $6 \cdot 7^n - 2 \cdot 3^n, 36 \cdot 7^n, and \ -4 \cdot 3^n$, it divides their sum, which is $6 \cdot 7^{n+1} - 2 \cdot 3^{n+1}$.